Government General Degree College, Chapra

Internal Assessment- 2nd Semester, 2023-24

Sub.- Physics (Major)

Paper: MATHEMATICAL PHYSICS

Total Marks: 15 Time: 40 minutes

Answer any three questions only:

1. (a) Plot the function $f(x) = x^2$ and its first derivative. 2
   (b) Find the series expansion of $\frac{1}{1-x}$. Mention its interval of convergence. 2+1

2. (a) Check whether $dw = 2xydx + x^2dy$ is an exact differential. 2
   (b) Solve the differential equation :- $(D^2 + 1)$y = cosx + $e^x sinx$ 3

3. (a) Solve the differential equation $(x+1)\frac{dy}{dx} - y = e^x (x+1)^2$. 3
   (b) Find the Taylor series expansion of lnx about $x = 2$. 2

4. Solve the equation –
   $y'' + 6y' + 8y = 0$, subject to the condition $y = 1$, $y' = 0$ at $x = 0$,
   where, $y' = \frac{dy}{dx}$ and $y'' = \frac{d^2y}{dx^2}$. 5

5. (a) Find a unit vector normal to $\vec{A} = 2\hat{\imath} + 4\hat{\jmath} - 5\hat{k}$ and $\vec{B} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k}$. 2
   (b) Using Stoke's law, prove that $\vec{\nabla} \times \vec{\nabla}\emptyset = 0$. 3

6. (a) The position vectors of three points *A, B* and *C* are $\vec{r_1} = \hat{\imath} + 2\hat{\jmath} + \hat{k}$, $\vec{r_2} = 3\hat{\imath} + 2\hat{\jmath} - 3\hat{k}$ and $\vec{r_3} = 2\hat{\imath} + 2\hat{\jmath} - 3\hat{k}$. Find the area of the triangle. 3
   (b) $\vec{\omega}$ is a constant vector and $\vec{r}$ is the position vector of a point. If $\vec{v} = \vec{\omega} \times \vec{r}$, then prove that $\vec{\nabla}.\vec{v} = 0$. 2

7. $If\ \vec{F} = x^2\hat{\imath} + y^2\hat{\jmath}$, then find the line integral $\int_c \vec{F}.d\vec{r}$ in the *x* - *y* plane along a line $y = x^2$ from *P(0,0)* to *Q(1,1)*. 5

8. (a) Find the eigenvalues of the matrix $\begin{pmatrix} 3 & i \\ -i & 3 \end{pmatrix}$. 2
   (b) Prove that the modulus of each characteristic root of a unitary matrix is unity. 3

9. (a) Using elementary operations, find inverse of matrix:
   $$A = \begin{pmatrix} -1 & 1 & 2 \\ 0 & 2 & 1 \\ -1 & 3 & 4 \end{pmatrix}$$ 3
   (b) Prove that the eigenvalue of a skew Hermitian is purely imaginary. 2